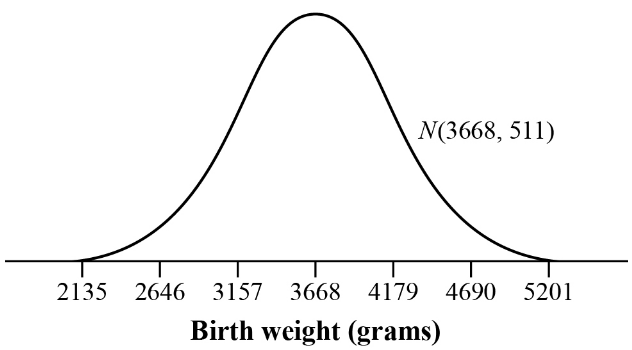
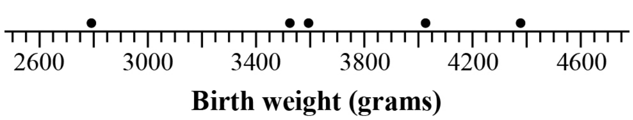
Chapter Review Exercises (page 466)

R7.1 The population is the set of all eggs shipped in one day. The sample consists of the 200 eggs examined. The parameter is the proportion $p = 0.03$ of eggs shipped that day that had salmonella. The statistic is the proportion $\hat{p} = \frac{9}{200} = 0.045$ of eggs in the sample that had salmonella.

R7.2 (a) A sketch of the population distribution is given below.



(b) Answers will vary. An example dotplot is given.



(c) The dot at 2750 represents one SRS of size 5 from this population where the sample range was 2750 grams.

R7.3 (a) The sample range is not an unbiased estimator of the population range. If it were unbiased, then the sampling distribution of the sample range would have 3417 (the actual range) as its mean. But, according to the graph, none of the 500 sample ranges were greater than 3000 (and certainly none were greater than 3417).
(b) If we want to reduce the variability of the sampling distribution of the sample range, we should take larger samples.

R7.4 (a) The mean is $\mu_{\hat{p}} = p = 0.15$.
(b) Because the sample size of $n = 1540$ is less than 10% of the population of all adults, the standard deviation of the sampling distribution of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\frac{0.15(0.85)}{1540}} = 0.0091$.
(c) Because $np = 1540(0.15) = 231$ and $n(1-p) = 1540(0.85) = 1309$ are both at least 10, the sampling distribution of $\hat{p}$ is approximately Normal.
(d) **Step 1: State the distribution and values of interest.** $\hat{p}$ has an approximately Normal distribution with mean 0.15 and standard deviation 0.0091. We want to find $P(0.13 \le \hat{p} \le 0.17)$.

Step 2: Perform calculations. Show your work. The standardized scores for the boundary values are $z = \frac{0.13 - 0.15}{0.0091} = -2.20$ and $z = \frac{0.17 - 0.15}{0.0091} = 2.20$. The desired probability is $P(-2.20 \le Z \le 2.20) = 0.9861 - 0.0139 = 0.9722$. *Using technology*: normalcdf(lower: 0.13, upper: 0.17, μ: 0.15, σ: 0.0091) = 0.9720. **Step 3: Answer the question.** There is a 0.9720 probability of obtaining a sample in which between 13% and 17% are joggers.

R7.5 (a) **Step 1: State the distribution and values of interest.** We have an SRS of size 100 drawn from a population in which the proportion who get a red light is $p = 0.30$, assuming the agents' claim is true. Thus, $\mu_{\hat{p}} = p = 0.30$. Because 100 is less than 10% of the population of travelers, $\sigma_{\hat{p}} = \sqrt{\frac{0.30(0.70)}{100}} = 0.0458$. Because $np = 100(0.30) = 30$ and $n(1-p) = 100(0.70) = 70$ are both at least 10, the sampling distribution of $\hat{p}$ can be approximated by a Normal distribution. We want to find $P(\hat{p} \le 0.20)$ using the $N(0.30, 0.0458)$ distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{0.20 - 0.30}{0.0458} = -2.18$. The desired probability is $P(Z \le -2.18) = 0.0146$. *Using technology*: normalcdf (lower: –1000, upper: 0.20, μ: 0.30, σ: 0.0458) = 0.0145. **Step 3: Answer the question.** There is a 0.0145 probability that 20% or fewer of the travelers get a red light.

(b) Because this is a small probability, there is convincing evidence against the agents' claim—it isn't plausible to get a sample proportion of travelers with a red light this small by chance alone.

R7.6 (a) **Step 1: State the distribution and values of interest.** X = WAIS score for a randomly selected individual follows a $N(100, 15)$ distribution and we want to find $P(X \ge 105)$. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{105 - 100}{15} = 0.33$ The desired probability is $P(Z \ge 0.33) = 1 - 0.6293 = 0.3707$. *Using technology*: normalcdf(lower: 105, upper: 1000, μ: 100, σ: 15) = 0.3694. **Step 3: Answer the question.** There is a 0.3694 probability of selecting an individual with a WAIS score of at least 105.

(b) The mean of the sampling distribution of $\bar{x}$ is $\mu_{\bar{x}} = \mu = 100$. Because the sample of size 60 is less than 10% of all adults, the standard deviation of the sampling distribution of $\bar{x}$ is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{60}} = 1.9365$.

(c) **Step 1: State the distribution and values of interest.** $\bar{x}$ = mean WAIS score follows a $N(100, 1.9365)$ distribution and we want to find $P(\bar{x} \ge 105)$. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{105 - 100}{1.9365} = 2.58$. The desired probability is $P(Z \ge 2.58) = 1 - 0.9951 = 0.0049$. *Using technology*: normalcdf(lower: 105, upper: 1000, μ: 100, σ: 1.9365) = 0.0049. **Step 3: Answer the question.** There is a 0.0049 probability of selecting a sample of 60 adults whose mean WAIS score is at least 105.

(d) The answer to (a) could be quite different depending on the shape of the population distribution. The answer to (b) would be the same because the mean and standard deviation do not depend on the shape of the population distribution. Because of the large sample size ($60 \ge 30$), the answer we gave for (c) would still be fairly reliable due to the central limit theorem.

R7.7 (a) Because the sample size is large ($n = 50 \geq 30$), the central limit theorem says that the distribution of $\bar{x}$ will be approximately Normal.
(b) **Step 1: State the distribution and values of interest.** The sampling distribution of $\bar{x}$ has mean $\mu_{\bar{x}} = \mu = 0.5$. Because 50 is less than 10% of all traps, the standard deviation is $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{0.7}{\sqrt{50}} = 0.0990$. Thus, $\bar{x}$ follows a $N(0.5, 0.0990)$ distribution and we want to find P($\bar{x} \geq 0.6$). **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \dfrac{0.6 - 0.5}{0.0990} = 1.01$. The desired probability is $P(Z \geq 1.01) = 1 - 0.8438 =$ 0.1562. *Using technology*: normalcdf(lower: 0.6, upper: 1000, μ : 0.5, σ : 0.0990) = 0.1562.
Step 3: Answer the question. There is a 0.1562 probability that the mean number of moths is greater than or equal to 0.6.
(c) No. Because this probability is not small, it is plausible that the sample mean number of moths is this high by chance alone. There is not convincing evidence that the moth population is getting larger in their state.